Homework exercise 3 – Simple Harmonic Motion

Total = 32 marks

Question 1:

Marks

Car engines use the ignition of fuel to release energy which moves the pistons up and down, causing the crankshaft to rotate.

The vertical motion of the piston approximates to simple harmonic motion.

Figure 8 shows different positions of a piston in a car engine.

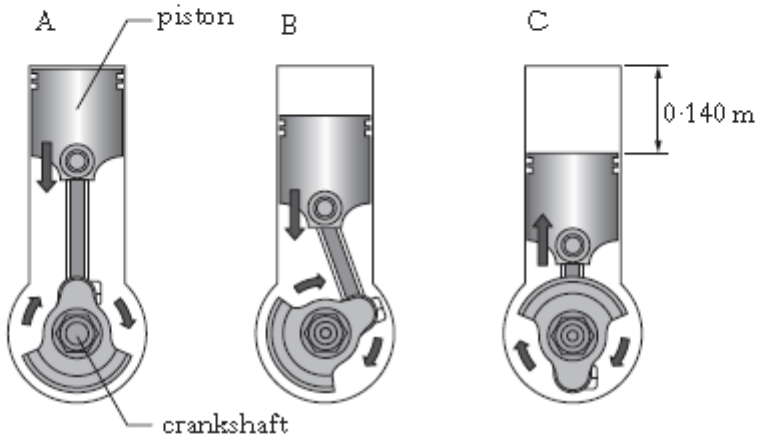

Figure 8

(*a*) Define *simple harmonic motion*. **1**

(*b*) Determine the amplitude of the motion. **1**

(*c*) In this engine the crankshaft rotates at 1500 revolutions per minute and the piston has a total mass of 1·40 kg.

(i) Calculate the maximum acceleration of the piston. **3**

(ii) Calculate the maximum kinetic energy of the piston. **2**

(7)

Question 2:

Marks

(*a*) A mass is suspended from a spring and is at rest.

The mass is displaced 20 mm from its rest position, as shown in Figure 7.

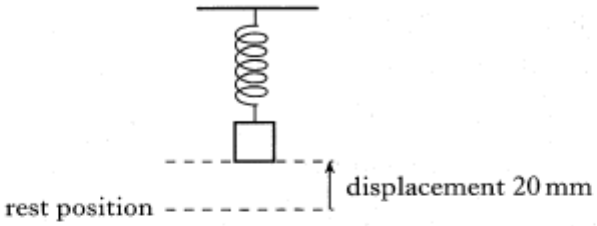

Figure 7

The mass is released.

A graph of the displacement y of the mass against time t is shown in Figure 8.

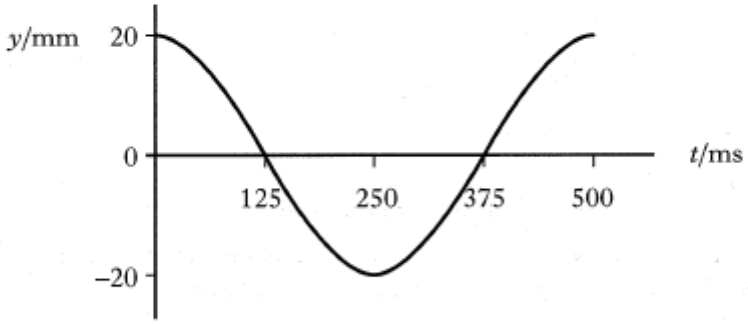

Figure 8

(i) Show, by calculation, that

$$\frac{d^2y}{dt^2} = -158y.$$

(ii) Sketch a graph of the velocity of the mass against time for the first period of the oscillation. Numerical values are required on both axes. **5**

(*b*) An object has a periodic motion and oscillates between A and B as shown in Figure 9.

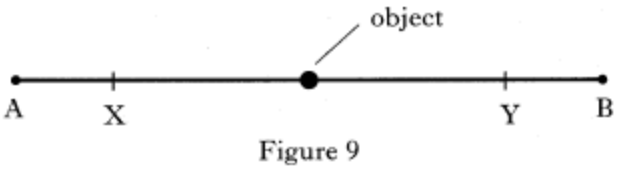

Figure 9

Between points X and Y the object moves with constant speed. Explain **fully** why the motion of the object cannot be described as simple harmonic. **2**

(7)

Question 3:

Marks

A "saucer" swing consists of a bowl shaped seat of mass 1·2 kg suspended by four ropes of negligible mass as shown in Figure 7A.

Figure 7A

When the empty seat is pulled back slightly from its rest position and released its motion approximates to simple harmonic motion.

(*a*) Define the term *simple harmonic motion*. **1**

(*b*) The acceleration-time graph for the seat with no energy loss is shown in Figure 7B.

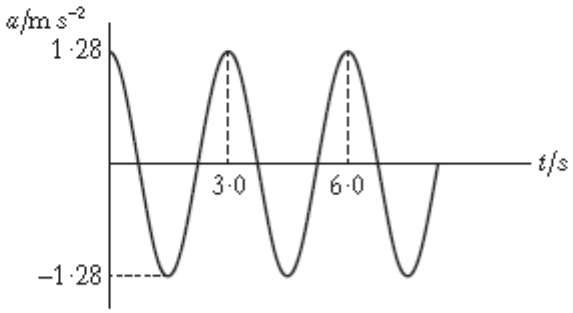

Figure 7B

(i) Show that the amplitude of the motion is 0·29 m. **3**

(ii) Calculate the velocity of the seat when its displacement is 0·10 m. **2**

(*c*) Calculate the displacement of the seat when the kinetic energy and potential energy are equal. **3**

(9)

Question 4:

Marks

4. A mass of 0·40 kg is suspended from a spring as shown in Figure 2. The mass is then displaced vertically and released. Its subsequent motion is recorded using a motion sensor linked to a computer.

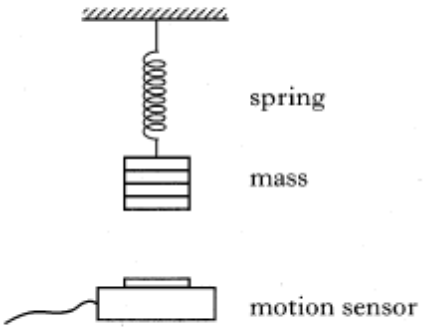

Figure 2

The mass moves with simple harmonic motion. The displacement-time graph of the mass is shown in Figure 3.

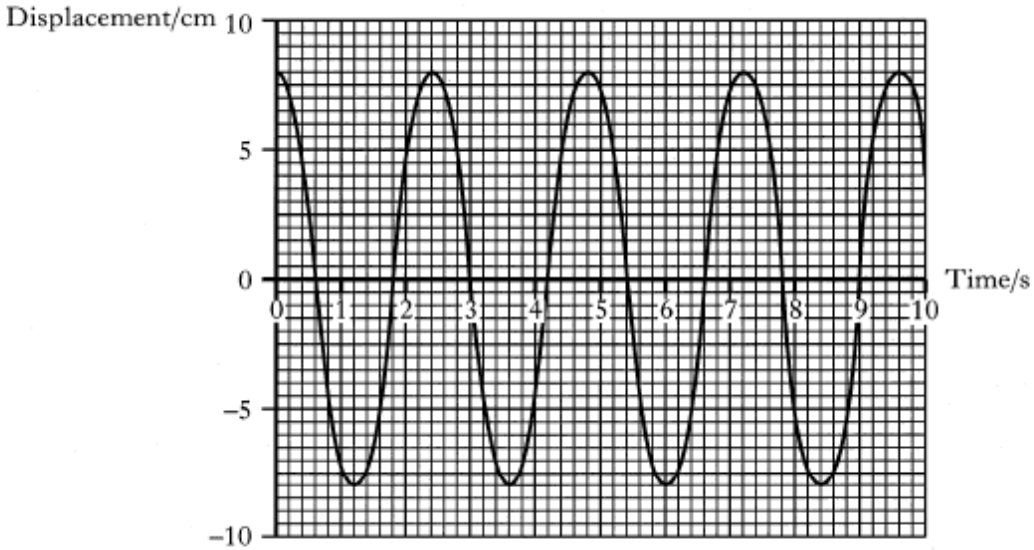

Figure 3

(*a*) Find:

(i) the amplitude of the oscillation;

(ii) the period of the oscillation. **2**

(*b*) Using the values from part (*a*), obtain an expression, in the form $y = A\cos\omega t$, for the vertical displacement y of the mass. **2**

(*c*) (i) Using the solution to part (*b*), derive an expression which gives the relationship between the acceleration a of the mass and time t.

(ii) Calculate the maximum kinetic energy of the mass. **5**

(9)